P1

Management Accounting

It's a relatively simple process to calculate the optimal manufacturing mix when two products are subject to one limiting factor, but P1 candidates need to know how to find the best combination in cases where two or more resources are restricted. This is where linear programming comes into its own

By Cathy Shirley ACMA, CGMA

You're probably already familiar with production planning in scenarios where there's a scarce resource or limiting factor, which usually takes the form of a shortage of raw materials or labour hours. Production schedules are best determined using the limiting or key factor technique in such cases. But, when a production planning scenario presents more than one scarce resource – a shortage of labour hours *and* raw materials, for instance – linear programming is the method to apply.

This seven-step procedure entails plotting a graph. Although students won't be required to draw one in the P1 exam, linear programming is still a key part of the syllabus. You must therefore understand the full sequence to follow before attempting any objective test questions on the topic, which will test your knowledge in a variety of ways.

Let's consider the following scenario involving a firm called the Stationery Company. It makes two luxury products: a fountain pen and a ballpoint pen. Demand for both is high. The pens are created from the same unique raw material and are crafted by hand. This work requires a mix of skilled and unskilled labour. We have the following information concerning their respective contributions:

	Fountain pen ($)	**Ballpoint pen ($)**
Selling price per unit	131	49
Raw material at $5/kg	(15)	(10)
Skilled labour at $10/hour	(50)	(20)
Unskilled labour at $4/hour	(16)	(4)
Variable overhead at $5/hour	(15)	(5)
Contribution per unit	35	10

The maximum amount of raw material available each week is 120kg, which represents a shortage. There are five unskilled workers, who can each work 20 hours a week, and four skilled workers, who can each put in 35 hours a week.

The priority for a business is to maximise shareholder wealth, which requires a comprehensive strategy involving both short- and long-term objectives. Maximising profit is one such objective. In the absence of other information, we can assume that this is the guiding principle behind optimising production. The company's fixed costs will not change in the short term, which means that its profit will be maximised by obtaining the highest total contribution possible from its products.

In pursuit of the optimum production plan, the Stationery Company's production manager faces a problem in that there are insufficient resources to make unrestricted amounts of both products. Variable overheads are not specifically mentioned in the scenario provided, so we can assume that there are no scarce resources in this area. But only 120kg of raw material and only 140 skilled and 100 unskilled labour hours are available per week. Given that raw material and labour hours are both limited resources, we need to work out the mix of products that optimises the total contribution by going through the following seven-step process.

1. Define the variables

The variables we are trying to decide here are the quantities of each pen type to produce. As we'll be using graphs, it's easier to name these unknowns X and Y so as to reflect the x and y axes. Let's say that X is the number of fountain pens and Y is the number of ballpoint pens.

2. Define the objective function (the scenario objective)

In most short-term planning decisions, the objective will be to maximise the contribution. An alternative question could require you to minimise costs. From the product information provided in the scenario, we know that each fountain pen earns $35 of contribution and each ballpoint pen earns $10 of contribution. We want the optimum mix of X and Y to earn as much contribution as possible. This overall aim can be stated algebraically as: Max $C = 35X + 10Y$, which is known as the objective function.

3. Define the constraints

This step requires us to express each production constraint in the form of an equation, stated in terms of X and Y. Using the information provided in the scenario, the constraint relating to the raw material will be $3X + 2Y \leq 120$.

In this equation, 120 represents the total amount of raw material available each week in kilos. It denotes that 3kg of raw material is required for every unit of X produced and that 2kg of material is required for every unit of Y produced. The total must be equal to (or less than) 120, because this is the maximum amount available.

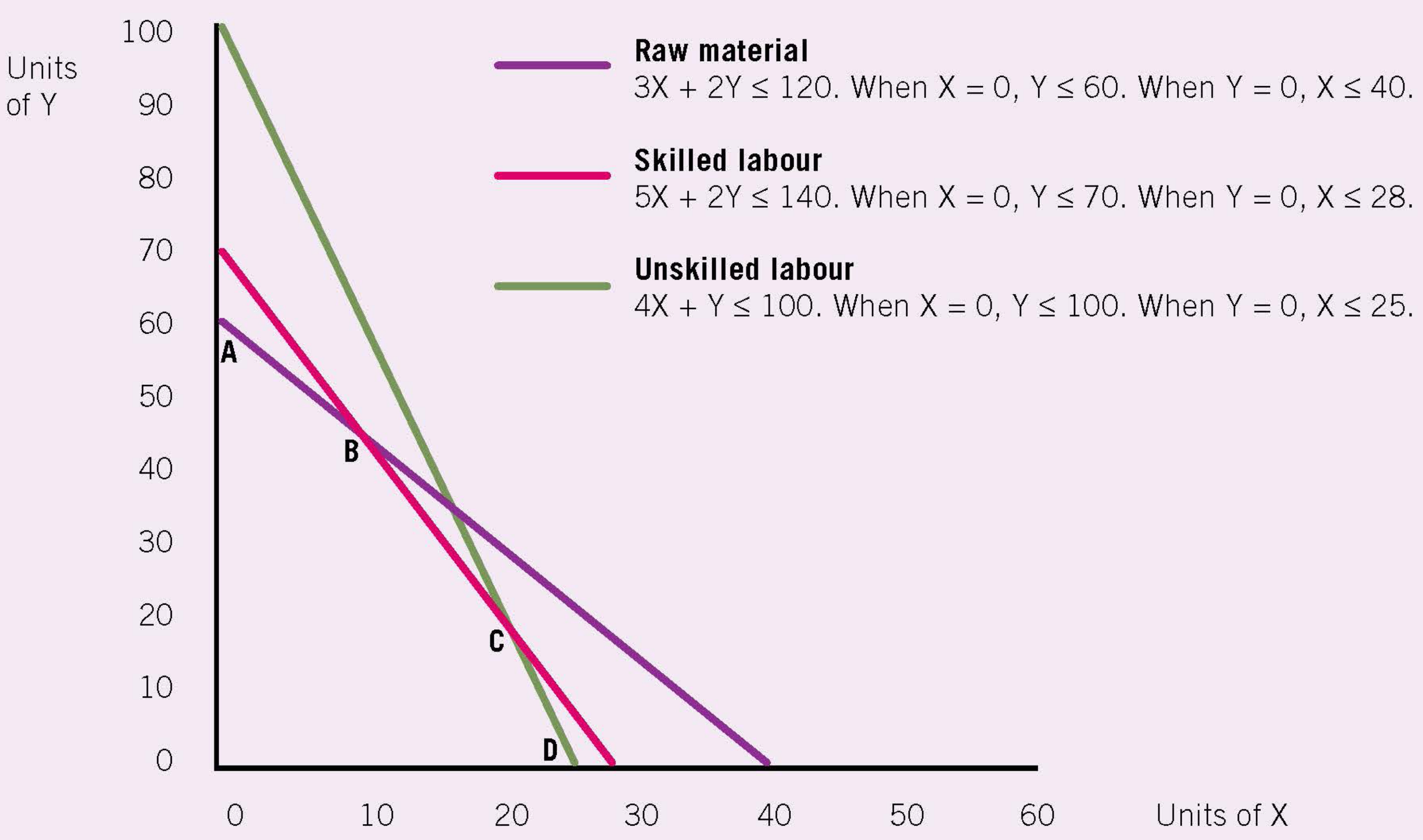
The other constraints can be expressed in a similar way:

- Skilled labour: $5X + 2Y \leq 140$.
- Unskilled labour: $4X + Y \leq 100$.

4. Plot the constraints as lines on a graph

For each formula, we use the method of equating one variable to zero in order to find the other variable's intercept – ie, we find the point at which the line hits the Y axis when X equals zero and then the point at which the line hits the X axis when Y equals zero. This provides two co-ordinates, enabling us to join them with a straight line on the graph below for each constraint.

Linear programming graph for the Stationery Company's production of pens X and Y



5. Label the feasible region

Any production combination within the area on the graph bounded by the points labelled A, B, C, D and the origin will be possible, given all the resource constraints. This area is known as the feasible region. The origin represents the point at which no products are made, so this will clearly not maximise the contribution. The point of optimal contribution must lie at one of the extremes of the region – ie, at corner A, B, C or D. When we find this point, it will tell the Stationery Company the contribution-maximising mix of fountain pens and ballpoint pens it should produce each week while all the constraints remain.

6. Find the optimum point using the iso-contribution line

To determine which corner of the feasible region represents the optimal contribution point, we need to plot the objective function – in this case, it's Max C = 35X + 10Y, which we established in the second step – on the graph in the form of an iso-contribution line.

Because Max C = 35X + 10Y has three unknowns, we need to assign an arbitrary value for C to plot this line. Any value for C can be used, but a common method is to multiply the product of the coefficients of X and Y by a common factor. This results in co-ordinates that are easier to plot. In this case let's multiply the X and Y co-ordinate values by 2, so that C = 35 x 10 x 2 = 700. So the objective function becomes 700 = 35X + 10Y, which we then plot on the graph using the same approach that we used in the fourth step.

All points on the resultant iso-contribution line represent combinations of X and Y that will yield the same total contribution. In this case, it's $700. You may wish to plot more than one iso-contribution line using different values for C. They will always be parallel – it's the slope of the line that matters here.

Next we place a ruler along the iso-contribution line we have plotted and slide it outwards away from the origin, keeping it parallel to the line. The dotted line on the left on the graph below is the original iso-contribution line and the one on the right represents the final position of the ruler at the point of its last contact with the feasible region.

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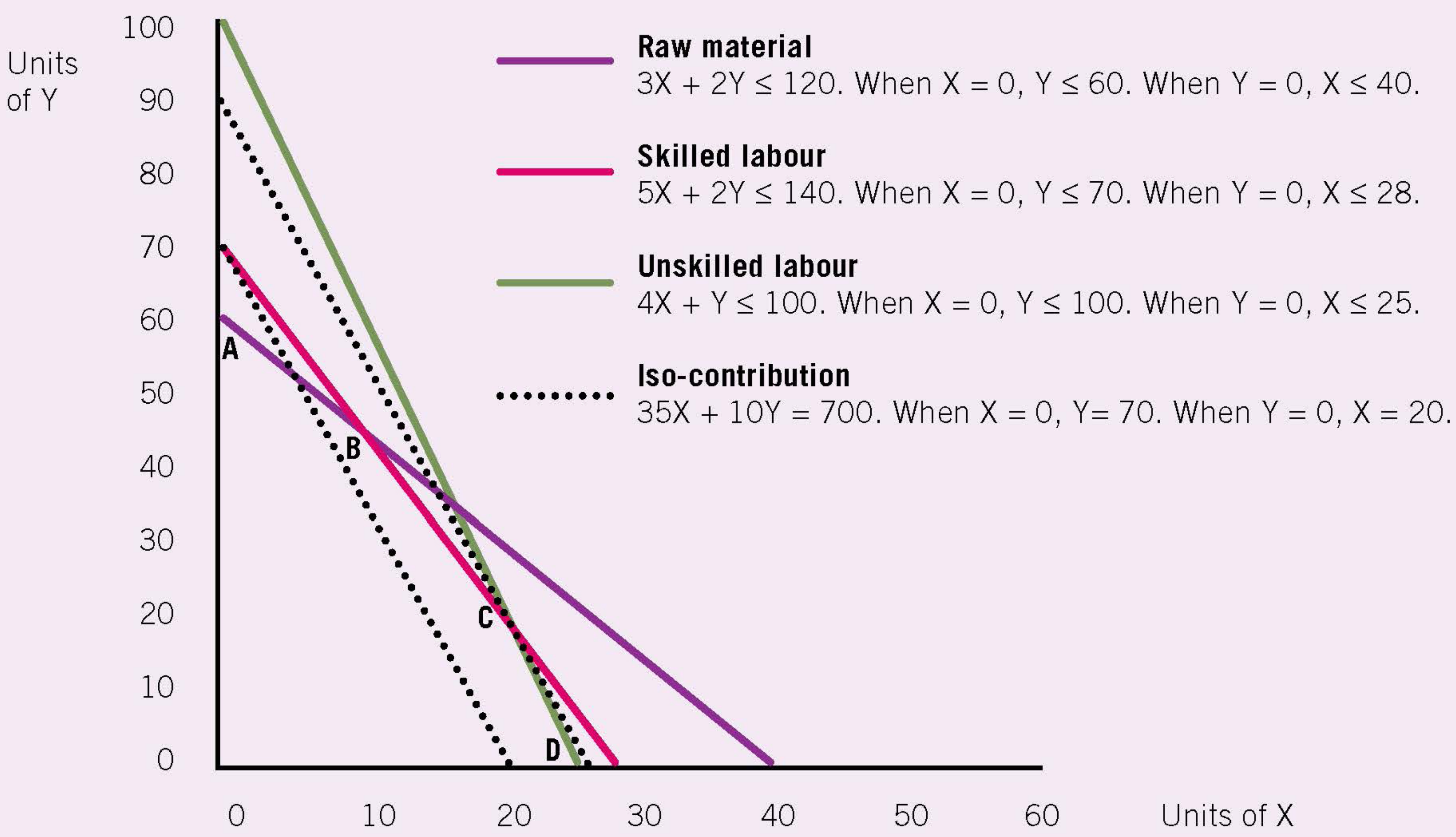

This point of optimum contribution, C, appears to be where X = 20 and Y = 20 (although we can confirm this in next step). The combination of 20 fountain pens and 20 ballpoint pens should maximise the total weekly contribution – and the company should have enough raw material and hours of skilled and unskilled labour to manufacture them.

7. Confirm optimal solution with simultaneous equations

The two lines that intersect at the optimum point indicate the binding constraints, which tell us which equations to

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use in this last step. At point C these are skilled labour and unskilled labour. A lack of these two resources is what is preventing the company from producing more units and thereby increasing their total contribution.

The two relevant equations are therefore:

- Skilled labour: $5X + 2Y = 140$.
- Unskilled labour: $4X + Y = 100$.

If we multiply the entire second equation by 2, it becomes $8X + 2Y = 200$. Now that both equations contain the common term 2Y, we can eliminate Y and find X by subtracting the first equation from the second as follows:

$(8X + 2Y) - (5X + 2Y) = 200 - 140$

$3X = 60$

$X = 20$

We can now go back to either of the original equations and replace X with 20. For example:

$(5 \times 20) + 2Y = 140$

$2Y = 140 - 100$

$Y = 20$

This confirms that the optimum weekly production mix is 20 fountain pens and 20 ballpoint pens as long as the resource constraints remain. It represents the maximum total contribution possible, given the limited amount of raw material and hours of skilled and unskilled labour available to the company.

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